

A Positive Answer to Erdős Problem 74 Would Imply a Positive Answer to Problem 750

May 1, 2026

Abstract

We record a short reduction between two open problems listed on erdosproblems.com. A positive answer to Erdős Problem 74, the edge-bipartisation version of the almost-bipartite large-chromatic problem, would imply a positive answer to Problem 750, the independent-set version. The proof is the elementary endpoint-selection inequality

$$\alpha(H) \geq \frac{|V(H)| - \beta(H)}{2},$$

where $\beta(H)$ is the minimum number of edges whose deletion makes H bipartite.

1 The two problems

Problem 74 asks whether, for every function $g(n) \rightarrow \infty$ arbitrarily slowly, there is a graph G of infinite chromatic number such that every finite subgraph on n vertices can be made bipartite by deleting at most $g(n)$ edges [EP74].

Problem 750 asks whether, for every function $f(n) \rightarrow \infty$ arbitrarily slowly, there is a graph G of infinite chromatic number such that every finite subgraph on n vertices has an independent set of size at least

$$\frac{n}{2} - f(n).$$

See [EP750].

Both problems are open. Problem 74 is listed as open even for $g(n) = \sqrt{n}$ [EP74]. The case $f(n) = \varepsilon n$ of Problem 750 follows from Theorem 1 of Erdős, Hajnal and Szemerédi [EHS82]; see also the discussion on [EP750]. Rödl proved the corresponding constant- ε edge-deletion version for finite chromatic targets [Rö82]. The full formulation of Problem 750, with $f(n) \rightarrow \infty$ arbitrarily slowly, remains open [EP750].

2 The endpoint-selection inequality

For a finite graph H , define its edge-bipartisation number by

$$\beta(H) := \min\{|F| : F \subseteq E(H), H - F \text{ is bipartite}\}.$$

Lemma 1. *Let H be a finite graph on n vertices. Then*

$$\alpha(H) \geq \frac{n - \beta(H)}{2}.$$

Proof. Let $F \subseteq E(H)$ be a minimum bipartising set, so $|F| = \beta(H)$. Choose one endpoint from each edge of F , and let X be the set of chosen vertices. Then $|X| \leq |F| = \beta(H)$. Every edge of F is incident to X , so $H - X$ is a subgraph of $H - F$, and hence is bipartite. A bipartite graph on $n - |X|$ vertices has an independent set of size at least $(n - |X|)/2$. Therefore

$$\alpha(H) \geq \alpha(H - X) \geq \frac{n - |X|}{2} \geq \frac{n - \beta(H)}{2}.$$

□

This endpoint-selection argument is closely related to the comparison made in Section 3 of [EHS82], where Erdős, Hajnal and Szemerédi introduce the edge-deletion function f_G^3 and compare it with their bipartite induced-subgraph function. The lemma above is the independent-set form needed for the present reduction.

3 The reduction

Claim 1. *A positive answer to Erdős Problem 74 implies a positive answer to Problem 750.*

Proof. Let $f(n) \rightarrow \infty$ be given. Apply Problem 74 with

$$g(n) = 2f(n),$$

which also tends to infinity. We obtain a graph G of infinite chromatic number such that every n -vertex subgraph H satisfies

$$\beta(H) \leq 2f(n).$$

By the lemma,

$$\alpha(H) \geq \frac{n - \beta(H)}{2} \geq \frac{n - 2f(n)}{2} = \frac{n}{2} - f(n).$$

Thus G satisfies the requirement of Problem 750.

If one insists that the function used in Problem 74 be integer-valued, replace $2f(n)$ by $\lfloor 2f(n) \rfloor$, after changing finitely many initial values if necessary; this does not affect the asymptotic hypothesis $g(n) \rightarrow \infty$. □

4 Remarks

Remark 1 (One-way implication). The implication is one-way. A graph may have large independent sets in all finite subgraphs without being close to bipartite in edge-deletion distance. Thus Problem 750 does not obviously imply Problem 74.

Remark 2 (Induced versus non-induced subgraphs). The reduction is insensitive to the distinction between induced and non-induced subgraphs. If an edge-bipartisation bound holds for induced subgraphs $G[U]$, then for any subgraph H on the same vertex set U we have

$$\beta(H) \leq \beta(G[U])$$

and

$$\alpha(H) \geq \alpha(G[U]),$$

since deleting edges cannot decrease the independence number.

Remark 3 (Rates). More quantitatively, any progress on Problem 74 at a specific rate immediately gives the corresponding independent-set lower bound. For example, a graph satisfying

$$\beta(H) \leq g(n)$$

for every n -vertex subgraph H also satisfies

$$\alpha(H) \geq \frac{n}{2} - \frac{g(n)}{2}.$$

Thus Problem 74 is a sufficient master problem for Problem 750.

References

- [EP74] Thomas Bloom, *Erdős Problem 74*, erdosproblems.com. <https://www.erdosproblems.com/74>.
- [EP750] Thomas Bloom, *Erdős Problem 750*, erdosproblems.com. <https://www.erdosproblems.com/750>.
- [EHS82] P. Erdős, A. Hajnal, and E. Szemerédi, *On almost bipartite large chromatic graphs*, *Annals of Discrete Mathematics* **12** (1982), 117–123. DOI: 10.1016/S0304-0208(08)73497-2.
- [Rö82] V. Rödl, *Nearly bipartite graphs with large chromatic number*, *Combinatorica* **2** (1982), 377–383. DOI: 10.1007/BF02579434.